1. (a) Express $5 \cos x - 3 \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(b) Hence, or otherwise, solve the equation

$$5 \cos x - 3 \sin x = 4$$

for $0 \leq x < 2\pi$, giving your answers to 2 decimal places.

(5)
(Total 9 marks)

á – their 0.27), rather than applying the correct method of (2ð – their principal angle – their *á*).
Premature rounding caused a significant number of candidates to lose at least 1 accuracy mark, notably with a solution of 0.28^c instead of 0.27^c.

2. Solve

$$\text{cosec}^2\, 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(Total 7 marks)

3. (a) Use the identity $\cos^2 \theta + \sin^2 \theta = 1$ to prove that $\tan^2 \theta = \sec^2 \theta - 1$.

(2)

(b) Solve, for $0 \leq \theta < 360°$, the equation

$$2 \tan^2 \theta + 4 \sec \theta + \sec^2 \theta = 2$$

(6)
(Total 8 marks)

4. (a) Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$, to show that

$$\cos 2A = 1 - 2\sin^2 A$$

(2)

The curves C_1 and C_2 have equations

$$C_1: y = 3\sin 2x$$

$$C_2: y = 4\sin^2 x - 2\cos 2x$$

(b) Show that the x-coordinates of the points where C_1 and C_2 intersect satisfy the equation

$$4\cos 2x + 3\sin 2x = 2$$

(3)

(c) Express $4\cos 2x + 3\sin 2x$ in the form $R\cos(2x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$, giving the value of α to 2 decimal places.

(3)

(d) Hence find, for $0 \leq x < 180°$, all the solutions of

$$4\cos 2x + 3\sin 2x = 2$$

giving your answers to 1 decimal place.

(4)
(Total 12 marks)

5. (a) Write down $\sin 2x$ in terms of $\sin x$ and $\cos x$.

(1)

(b) Find, for $0 < x < \pi$, all the solutions of the equation

$$\operatorname{cosec} x - 8\cos x = 0$$

giving your answers to 2 decimal places.

(5)
(Total 6 marks)

6. (a) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3 \sin\theta - 4 \sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8 \sin^3\theta - 6 \sin \theta + 1 = 0.$$

Give your answers in terms of π.

(5)

(b) Using $\sin(\theta - \alpha) = \sin\theta\cos\alpha - \cos\theta\sin\alpha$, or otherwise, show that

$$\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2}).$$

(4)
(Total 13 marks)

7. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \cot^2\theta \equiv \text{cosec}^2\theta$.

(2)

(b) Solve, for $0 \leq \theta < 180°$, the equation

$$2\cot^2\theta - 9\text{cosec}\theta = 3,$$

giving your answers to 1 decimal place.

(6)
(Total 8 marks)

8. (a) Using $\sin^2\theta + \cos^2\theta \equiv 1$, show that $\text{cosec}^2\theta - \cot^2\theta \equiv 1$.

(2)

(b) Hence, or otherwise, prove that

$$\text{cosec}^4\theta - \cot^4\theta \equiv \text{cosec}^2\theta + \cot^2\theta.$$

(2)

(c) Solve, for $90° < \theta < 180°$,

$$\text{cosec}^4\theta - \cot^4\theta = 2 - \cot\theta.$$

(6)
(Total 10 marks)

9. (a) Show that

(i) $$\frac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \qquad x \neq (n - \tfrac{1}{4})\pi, \; n \in \mathbb{Z}$$

(2)

(ii) $$\tfrac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \tfrac{1}{2}$$

(3)

(b) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(c) Solve, for $0 \leq \theta \leq 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)
(Total 12 marks)

10. $f(x) = 12 \cos x - 4 \sin x$.

Given that $f(x) = R \cos(x + \alpha)$, where $R \geq 0$ and $0 \leq \alpha \leq 90°$,

(a) find the value of R and the value of α.

(4)

(b) Hence solve the equation

$$12 \cos x - 4 \sin x = 7$$

for $0 \leq x \leq 360°$, giving your answers to one decimal place.

(5)

(c) (i) Write down the minimum value of $12 \cos x - 4 \sin x$.

(1)

(ii) Find, to 2 decimal places, the smallest positive value of x for which this minimum value occurs.

(2)
(Total 12 marks)

11. (a) Given that $2 \sin(\theta + 30)° = \cos(\theta + 60)°$, find the exact value of $\tan \theta°$.

(5)

(b) (i) Using the identity $\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2 \sin^2 A.$$

(2)

(ii) Hence solve, for $0 \leq x < 2\pi$,

$$\cos 2x = \sin x,$$

giving your answers in terms of π.

(5)

(iii) Show that $\sin 2y \tan y + \cos 2y \equiv 1$, for $0 \leq y < \frac{1}{2}\pi$.

(3)
(Total 15 marks)

12. (a) Given that $\sin^2\theta + \cos^2\theta \equiv 1$, show that $1 + \tan^2\theta \equiv \sec^2\theta$.

(2)

(b) Solve, for $0 \le \theta < 360°$, the equation

$$2\tan^2\theta + \sec\theta = 1,$$

giving your answers to 1 decimal place.

(6)
(Total 8 marks)

13. (a) Using the identity $\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(b) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(c) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

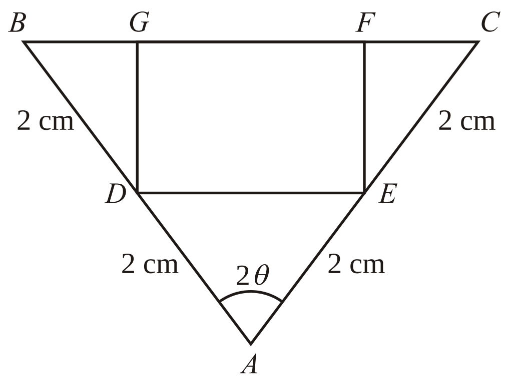
(d) Hence, for $0 \le \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)
(Total 15 marks)

14.



This diagram shows an isosceles triangle ABC with $AB = AC = 4$ cm and $\angle BAC = 2\theta$.

The mid-points of AB and AC are D and E respectively. Rectangle $DEFG$ is drawn, with F and G on BC. The perimeter of rectangle $DEFG$ is P cm.

(a) Show that $DE = 4 \sin \theta$.

(2)

(b) Show that $P = 8 \sin\theta + 4 \cos\theta$.

(2)

(c) Express P in the form $R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

Given that $P = 8.5$,

(d) find, to 3 significant figures, the possible values of θ.

(5)
(Total 13 marks)

15. (a) Sketch, on the same axes, in the interval $0 \le x \le 180$, the graphs of

$$y = \tan x° \text{ and } y = 2 \cos x°,$$

showing clearly the coordinates of the points at which the graphs meet the axes.

(4)

(b) Show that $\tan x° = 2 \cos x°$ can be written as

$$2 \sin^2 x° + \sin x° - 2 = 0.$$

(3)

(c) Hence find the values of x, in the interval $0 \le x \le 180$, for which $\tan x° = 2 \cos x°$.

(4)
(Total 11 marks)

16. (i) (a) Express ($12 \cos \theta - 5 \sin \theta$) in the form $R \cos (\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Hence solve the equation

$$12 \cos \theta - 5 \sin \theta = 4,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(3)

(ii) Solve

$$8 \cot \theta - 3 \tan \theta = 2,$$

for $0 < \theta < 90°$, giving your answer to 1 decimal place.

(5)
(Total 12 marks)

17. (i) Given that $\cos(x + 30)° = 3 \cos(x - 30)°$, prove that $\tan x° = -\frac{\sqrt{3}}{2}$.

(5)

(ii) (a) Prove that $\frac{1 - \cos 2\theta}{\sin 2\theta} \equiv \tan \theta$.

(3)

(b) Verify that $\theta = 180°$ is a solution of the equation $\sin 2\theta = 2 - 2 \cos 2\theta$.

(1)

(c) Using the result in part (a), or otherwise, find the other two solutions, $0 < \theta < 360°$, of the equation $\sin 2\theta = 2 - 2 \cos 2\theta$.

(4)
(Total 13 marks)

18. (a) Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$.

(4)

(b) Show that the equation $\sec x + \sqrt{3}\operatorname{cosec} x = 4$ can be written in the form

$$\sin x + \sqrt{3}\cos x = 2\sin 2x.$$

(3)

(c) Deduce from parts (a) and (b) that $\sec x + \sqrt{3}\operatorname{cosec} x = 4$ can be written in the form

$$\sin 2x - \sin(x + 60°) = 0.$$

(1)

(d) Hence, using the identity $\sin X - \sin Y = 2\cos\frac{X+Y}{2}\sin\frac{X-Y}{2}$, or otherwise, find the values of x in the interval $0 \le x \le 180°$, for which $\sec x + \sqrt{3}\operatorname{cosec} x = 4$.

(5)
(Total 13 marks)

19. On separate diagrams, sketch the curves with equations

(a) $y = \arcsin x, \quad -1 \le x \le 1,$

(b) $y = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{3}$, stating the coordinates of the end points of your curves in each case.

(4)

Use the trapezium rule with five equally spaced ordinates to estimate the area of the region bounded by the curve with equation $y = \sec x$, the x-axis and the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, giving your answer to two decimal places.

(4)
(Total 8 marks)

20. (a) Prove that for all values of x,

$$\sin x + \sin (60° - x) \equiv \sin (60° + x).$$

(4)

(b) Given that $\sin 84° - \sin 36° = \sin \alpha°$, deduce the exact value of the acute angle α.

(2)

(c) Solve the equation

$$4 \sin 2x + \sin (60° - 2x) = \sin (60° + 2x) - 1$$

for values of x in the interval $0 \leq x < 360°$, giving your answers to one decimal place.

(5)
(Total 11 marks)

21. Find, giving your answers to two decimal places, the values of w, x, y and z for which

(a) $e^{-w} = 4$,

(2)

(b) $\arctan x = 1$,

(2)

(c) $\ln (y + 1) - \ln y = 0.85$

(4)

(d) $\cos z + \sin z = \frac{1}{3}$, $-\pi < z < \pi$.

(5)
(Total 13 marks)

22. In a particular circuit the current, I amperes, is given by

$$I = 4 \sin \theta - 3 \cos \theta, \qquad \theta > 0,$$

where θ is an angle related to the voltage.

Given that $I = R \sin (\theta - \alpha)$, where $R > 0$ and $0 \leq \alpha < 360°$,

(a) find the value of R, and the value of α to 1 decimal place.

(4)

(b) Hence solve the equation $4 \sin \theta - 3 \cos \theta = 3$ to find the values of θ between 0 and 360°.

(5)

(c) Write down the greatest value for I.

(1)

(d) Find the value of θ between 0 and 360° at which the greatest value of I occurs.

(2)
(Total 12 marks)

1. (a) $5\cos x - 3\sin x = R\cos(x+\alpha),\ R > 0,\ 0 < x < \frac{\pi}{2}$

$5\cos x - 3\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

Equate $\cos x$: $5 = R\cos\alpha$

Equate $\sin x$: $3 = R\sin\alpha$

$R = \sqrt{5^2+3^2}\,; = \sqrt{34}\{= 5.83095...\}$	$R^2 = 5^2 + 3^2$	M1;	
	$\sqrt{34}$ or awrt 5.8	A1	
$\tan\alpha = \frac{3}{5} \Rightarrow \alpha = 0.5404195003...^c$	$\tan\alpha = \pm\frac{3}{5}$ or $\tan\alpha = \pm\frac{5}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{5}{\text{their } R}$ or	M1	
	α = awrt 0.54 or α = awrt 0.17π or or $\alpha = \frac{\pi}{\text{awrt } 5.8}$	A1	4

Hence, $5\cos x - 3\sin x = \sqrt{34}\cos(x + 0.5404)$

(b) $5\cos x - 3\sin x = 4$

$\sqrt{34}\cos(x + 0.5405) = 4$

$\cos(x + 0.5404) = \frac{4}{\sqrt{34}}\{= 0.68599...\}$	$\cos(x \pm \text{their } \alpha) = \frac{4}{\text{their } R}$	M1	
$(x + 0.5404) = 0.814826916...^c$	For applying $\cos^{-1}\left(\frac{4}{\text{their } R}\right)$	M1	
$x = 0.2744...^c$	awrt 0.27^c	A1	
$(x + 0.5404) = 2\pi - 0.814826916...^c$ $\{= 5.468358...^c\}$	2π – their 0.8148	ddM1	
$x = 4.9279...^c$	awrt 4.93^c	A1	5

Hence, $x = \{0.27, 4.93\}$

Note

If there are any EXTRA solutions inside the range $0 \le x < 2\pi$, then withhold the final accuracy mark if the candidate would otherwise score all 5 marks. Also ignore EXTRA solutions outside the range $0 \le x < 2\pi$.

[9]

2. $\text{cosec}^2 2x - \cot 2x = 1$, (eqn *)
$0 \le x \le 180°$

Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives	Writing down or using $\text{cosec}^2 2x = \pm 1 \pm \cot^2 2x$	M1
$1 + \cot^2 2x - \cot 2x = 1$	or $\text{cosec}^2\theta = \pm 1 \pm \cot^2\theta$	
$\underline{\cot^2 2x - \cot 2x} = 0$ or $\cot^2 2x = \cot 2x$	For either $\underline{\cot^2 2x - \cot 2x}\{=0\}$ or $\cot^2 2x = \cot 2x$	A1
$\cot 2x(\cot 2x - 1) = 0$ or $\cot 2x = 1$	Attempt to factorise or solve a quadratic (See rules for factorising quadratics) or cancelling out cot 2x from both sides.	dM1
$\cot 2x = 0$ or $\cot 2x = 1$	Both $\cot 2x = 0$ and $\cot 2x = 1$.	A1
$\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$	Candidate attempts to divide at least one of their principal anglesby 2. This will be usually implied by seeing$x = 22.5$ resulting from $\cot 2x = 1$.	ddM1
$\Rightarrow x = 45, 135$		
$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$		
$\Rightarrow x = 22.5, 112.5$		
Overall, $x = \{22.5, 45, 112.5, 135\}$	***Both*** $x = 22.5$ and $x = 112.5$	A1
	Both $x = 45$ and $x = 135$	B1

If there are any EXTRA solutions inside the range $0 \le x \le 180°$ and the candidate would otherwise score FULL MARKS then withhold the final accuracy mark (the sixth mark in this question).

Also ignore EXTRA solutions outside the range $0 \le x \le 180°$.

[7]

3. (a) $\cos^2\theta + \sin^2\theta = 1$ ($\div \cos^2\theta$)

$\underline{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}}$	Dividing $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$ to give underlined equation.	M1	
$1 + \tan^2\theta = \sec^2\theta$			
$\tan^2\theta = \sec^2\theta - 1$ (as required) **AG**	Complete proof.	A1 cso	2

No errors seen.

(b) $2\tan^2\theta + 4\sec\theta + \sec^2\theta = 2$, (eqn *) $0 \le \theta < 360°$

$2(\sec^2\theta - 1) + 4\sec\theta + \sec^2\theta = 2$	Substituting $\tan^2\theta = \sec^2\theta - 1$ into eqn * to get a quadratic in $\sec\theta$ only	M1	
$2\sec^2\theta - 2 + 4\sec\theta + \sec^2\theta = 2$			
$\underline{3\sec^2\theta + 4\sec\theta - 4} = 0$	Forming a three term "one sided" quadratic expression in $\sec\theta$.	M1	
$(\sec\theta + 2)(3\sec\theta - 2) = 0$	Attempt to factorise or solve a quadratic.	M1	
$\sec\theta = -2$ or $\sec\theta = \frac{2}{3}$			
$\frac{1}{\cos\theta} = -2$ or $\frac{1}{\cos\theta} = \frac{2}{3}$			
$\underline{\cos\theta = -\frac{1}{2}}$; or $\cos\theta = \frac{3}{2}$	$\underline{\cos\theta = -\frac{1}{2}}$;	A1;	
$\alpha = 120°$ or α = no solutions			
$\theta_1 = \underline{120°}$	$\underline{120°}$	<u>A1</u>	
$\theta_2 = 240°$	$\underline{240°}$ or $\theta_2 = 360° - \theta_1$ when solving using $\cos\theta = ...$	B1ft	6
$\theta = \{120°, 240°\}$	Note the final A1 mark has been changed to a B1 mark.		

[8]

4. (a)

$A = B \Rightarrow \cos(A + A) = \cos 2A = \underline{\cos A\cos A - \sin A\sin A}$	Applies $A = B$ to $\cos(A + B)$ to give the underlined equation or $\cos 2A = \underline{\cos^2 A - \sin^2 A}$	M1	
$\cos 2A = \cos^2 A - \sin^2 A$ and $\cos^2 A + \sin^2 A = 1$ gives			
$\underline{\cos 2A} = 1 - \sin^2 A - \sin^2 A = \underline{1 - 2\sin^2 A}$ (as required) Complete proof, with a link between LHS and RHS. No errors seen.		A1 **AG**	2

(b)	$C_1 = C_2 \Rightarrow 3\sin 2x = 4\sin^2 x - 2\cos 2x$	Eliminating y correctly.	M1	
	$3\sin 2x = 4\left(\frac{1-\cos 2x}{2}\right) - 2\cos 2x$	Using result in part (a) to substitute for $\sin^2 x$ as $\frac{\pm 1 \pm \cos 2x}{2}$ or $k\sin^2 x$ as $k\left(\frac{\pm 1 \pm \cos 2x}{2}\right)$ to produce an equation in only double angles.	M1	
	$3\sin 2x = 2(1 - \cos 2x) - 2\cos 2x$			
	$3\sin 2x = 2 - 2\cos 2x - 2\cos 2x$			
	$3\sin 2x + 4\cos 2x = 2$	Rearranges to give correct result	A1 **AG**	3
(c)	$3\sin 2x + 4\cos 2x = R\cos(2x - \alpha)$			
	$3\sin 2x + 4\cos 2x = R\cos 2x\cos\alpha + R\sin 2x\sin\alpha$			
	Equate $\sin 2x$: $3 = R\sin\alpha$			
	Equate $\cos 2x$: $4 = R\cos\alpha$			
	$R = \sqrt{3^2 + 4^2}; = \sqrt{25} = 5$	$R = 5$	B1	
	$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 36.86989765...°$	$\sin\alpha = \pm\frac{3}{4}$ or $\tan\alpha = \pm\frac{4}{3}$ or $\sin\alpha = \pm\frac{3}{\text{their } R}$ or $\cos\alpha = \pm\frac{4}{\text{their } R}$	M1	
		awrt 36.87	A1	3
	Hence, $3\sin 2x + 4\cos 2x = 5\cos(2x - 36.87)$			
(d)	$3\sin 2x + 4\cos 2x = 2$			
	$5\cos(2x - 36.87) = 2$			
	$\cos(2x - 36.87) = \frac{2}{5}$	$\cos(2x \pm \text{their } \alpha) = \frac{2}{\text{their } R}$	M1	
	$(2x - 36.87) = 66.42182...°$	awrt 66	A1	
	$(2x - 36.87) = 360 - 66.42182...°$			
	Hence, $x = 51.64591...$o, $165.22409...°$	One of either awrt 51.6 or awrt 51.7 or awrt 165.2 or awrt 165.3	A1	
		Both awrt 51.6 AND awrt 165.2	A1	4
		If there are any EXTRA solutions inside the range $0 \le x < 180°$ then withhold the final accuracy mark. Also ignore EXTRA solutions outside the range $0 \le x < 180°$.		

[12]

5. (a) $\sin 2x = \underline{2 \sin x \cos x}$ B1 aef 1

(b) $\operatorname{cosec} x - 8 \cos x = 0,\ 0 < x < \pi$

$\frac{1}{\sin x} - 8\cos x = 0$ Using $\operatorname{cosec} x = \frac{1}{\sin x}$ M1

$\frac{1}{\sin x} = 8\cos x$

$1 = 8 \sin x \cos x$

$1 = 4(2 \sin x \cos x)$

$1 = 4 \sin 2x$

$\underline{\sin 2x = \tfrac{1}{4}}$ $\sin 2x = k$, where $-1 < k < 1$ and $k \neq 0$ M1

$\underline{\sin 2x = \tfrac{1}{4}}$ A1

Radians $2x = \{0.25268..., 2.88891...\}$

Degrees $2x = \{14.4775..., 165.5225...\}$

Radians $x = \{0.12634..., 1.44445...\}$

Degrees $x = \{7.23875..., 82.76124...\}$

Either arwt 7.24 or 82.76 or 0.13 or 1.44 or 1.45 or awrt 0.04π or awrt 0.46π. A1

Both $\underline{0.13}$ and $\underline{1.44}$ A1 cao 5

Solutions for the final two A marks must be given in x only.
If there are any EXTRA solutions inside the range $0 < x < \pi$ then withhold the final accuracy mark.
Also ignore EXTRA solutions outside the range $0 < x < \pi$.

[6]

6. (a) (i) $\sin 3\theta = \sin(2\theta + \theta)$

$= \sin 2\theta \cos\theta + \cos 2\theta \sin \theta$

$= 2\sin\theta \cos\theta.\cos\theta + (1 - 2\sin^2 \theta)\sin \theta$ M1 A1

$= 2\sin\theta\,(1 - \sin 2\theta) + \sin\theta - 2\sin^3\theta$ M1

$= 3\sin\theta - 4\sin^3\theta$ * cso A1 4

(ii) $8\sin^3\theta - 6\sin\theta + 1 = 0$

$-2\sin 3\theta + 1 = 0$ M1 A1

$\sin 3\theta = \frac{1}{2}$ M1

$3\theta \frac{\pi}{6}, \frac{5\pi}{6}$

$\theta = \frac{\pi}{18}, \frac{5\pi}{6}$ A1 A1 5

(b) $\sin 15° = \sin(60° - 45°) = \sin 60°\cos 45° - \cos 60°\sin 45°$ M1

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ M1 A1

$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ $*$ cso A1 4

Alternatives

① $\sin 15° = \sin(45° - 30°) = \sin 45°\cos 30° - \cos 45°\sin 30°$ M1

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ M1 A1

$= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ $*$ cso A1 4

② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$

$2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$

$\sin^2 15° = \frac{2 - \sqrt{3}}{4}$ M1 A1

$\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$ M1

Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ $*$ cso A1 4

[13]

7.	(a)	$\sin^2\theta + \cos^2\theta = 1$			
		$\div \sin^2\theta$ $\quad \dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1	
		$1 + \cot^2\theta = \operatorname{cosec}^2\theta$*	cso	A1	2
		Alternative			
		$1 + \cot^2\theta = 1 + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$		M1	
		$\operatorname{cosec}^2\theta$*	cso	A1	
	(b)	$2(\operatorname{cosec}^2\theta - 1) - 9\operatorname{cosec}\theta = 3$		M1	
		$2\operatorname{cosec}^2\theta - 9\operatorname{cosec}\theta - 5 = 0$ or $5\sin^2\theta + 9\sin\theta - 2 = 0$		M1	
		$(2\operatorname{cosec}\theta + 1)(\operatorname{cosec}\theta - 5) = 0$ or $(5\sin\theta - 1)(\sin\theta + 2) = 0$		M1	
		$\operatorname{cosec}\theta = 5$ or $\sin\theta = \dfrac{1}{5}$		A1	
		$\theta = 11.5°, 168.5°$		A1A1	6

[8]

8.	(a)	Dividing $\sin^2\theta + \cos^2\theta \equiv 1$ by $\sin^2\theta$ to give	M1	
		$\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} \equiv \dfrac{1}{\sin^2\theta}$		
		Completion: $1 + \cot^2\theta = \operatorname{cosec}^2 \Rightarrow \operatorname{cosec}^2\theta - \cot^2\theta \equiv 1$ AG	A1*	2
	(b)	$\operatorname{cosec}^4\theta - \cot^4\theta \equiv (\operatorname{cosec}^2\theta - \cot^2\theta)(\operatorname{cosec}^2\theta + \cot^2\theta)$	M1	
		$\equiv (\operatorname{cosec}^2\theta + \cot^2\theta)$ using (a) AG	A1*	2

*Using LHS = $(1 + \cot^2\theta)^2 - \cot^4\theta$, using (a) & elim. $\cot^4\theta$ M1, conclusion {using (a) again} A1**

Conversion to sines and cosines: needs $\dfrac{(1-\cos^2\theta)(1+\cos^2\theta)}{\sin^4\theta}$ *for M1*

(c)	Using (b) to form $\text{cosec}^2\theta + \cot^2\theta \equiv 2 - \cot\theta$	M1	
	Forming quadratic in $\cot\theta$	M1	
	$\Rightarrow 1 + \cot^2\theta + \cot^2\theta \equiv 2 - \cot\theta$ {using (a)}	M1	
	$2\cot^2\theta + \cot\theta - 1 = 0$	A1	
	Solving: $(2\cot\theta - 1)(\cot\theta + 1 = 0)$ to $\cot\theta =$	M1	
	$\left(\cot\theta = \frac{1}{2}\right)$ or $\cot\theta = -1$	A1	
	$\theta = 135°$ (or correct value(s) for candidate dep. on 3Ms)	A1ft	6
	Ignore solutions outside range *Extra "solutions" in range loses A1ft, but candidate may possibly have more than one "correct" solution.*		

[10]

9.	(a)	(i)	$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}$	M1	
			$= \frac{(\cos x + \sin x)(\cos x - \sin x)}{\cos x + \sin x}$		
			$= \cos x - \sin x$ **AG**	A1	2
		(ii)	$\frac{1}{2}(\cos 2x - \sin 2x) = \frac{1}{2}(2\cos^2 x - 1 - 2\sin x\cos x)$	M1, M1	
			$= \cos^2 x - \frac{1}{2} - \sin x\cos x$ **AG**	A1	3
	(b)		$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$		
			$\cos\theta(\cos\theta - \sin\theta) = \frac{1}{2}$	M1	
			$\cos^2\theta - \cos\theta\sin\theta = \frac{1}{2}$		
			$\frac{1}{2}(\cos 2\theta + 1) - \frac{1}{2}\sin 2\theta = \frac{1}{2}$	M1	
			$\frac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$		
			$\sin 2\theta = \cos 2\theta$ **AG**	A1	3

(c)	$\sin 2\theta = \cos 2\theta$			
	$\tan 2\theta = 1$		M1	
	$2\theta = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$		A1 for 1	
	$\theta = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$		M1 (4 solns)	
			A1	4

[12]

10.	(a)	$R\cos\alpha = 12$, $R\sin\alpha = 4$			
		$R = \sqrt{(12^2 + 4^2)} = \sqrt{160}$	Accept if just written down, awrt 12.6	M1 A1	
		$\tan\alpha = \frac{4}{12}$, $\Rightarrow$ $\alpha \approx 18.43°$	awrt 18.4°	M1, A1	4
	(b)	$\cos(x + \text{their } \alpha) = \frac{7}{\text{their } R}$ (≈ 0.5534)		M1	
		$x + \text{their } \alpha = 56.4°$	awrt 56°	A1	
		= ... , 303.6°	360° – their principal value	M1	
		x = °38.0, 285.2°	Ignore solutions out of range	A1, A1	5
		If answers given to more than 1 dp, penalise first time then accept awrt above.			
	(c)	(i) minimum value is $-\sqrt{160}$	ft their R	B1ft	
		(ii) $\cos(x + \text{their } \alpha) = -1$		M1	
		$x \approx 161.57°$	cao	A1	3

[12]

11.	(a)	$2\sin(\theta + 30)° = \cos(\theta + 60)°$		
		$2\sin\theta°\cos30° + 2\cos\theta°\sin30° = \cos\theta°\cos60° - \sin\theta°\sin60°$	B1B1	
		$\frac{2\sqrt{3}}{2}\sin\theta° + \frac{2}{2}\cos\theta° = \frac{1}{2}\cos\theta° - \frac{\sqrt{3}}{2}\sin\theta°$	M1	
		Finding $\tan\theta°$, $\tan\theta° = -\frac{1}{3\sqrt{3}}$ or equiva. Exact	M1,A1	5

(b) (i) Setting $A = B$ to give $\cos 2A = \cos^2 A - \sin^2 A$ M1

Correct completion:= $(1 - \sin^2 A) - \sin^2 A = 1 - 2\sin^2 A$ A1 2

Need to see intermediate step above for A1

(ii) Forming quadratic in $\sin x$ $[2\sin^2 x + \sin x - 1 = 0]$ M1

Solving $[(2\sin x - 1)(\sin x + 1) = 0$ or formula$]$ M1

$[\sin\theta = ½$ or $\sin\theta = -1]$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$; A1,A1ft

A1ft for $\pi - $ "α"

$\theta = \frac{3\pi}{2}$ A1 5

(iii) LHS = $2\sin y \cos y \frac{\sin y}{\cos y} + (1 - 2\sin^2 y)$ B1M1

B1 use of $\tan y = \frac{\sin y}{\cos y}$, *M1 forming expression in* $\sin y$, $\cos y$ *only*

Completion: $= 2\sin^2 y + (1 - 2\sin^2 y) = 1$ AG A1 3

[Alternative: LHS = $\frac{\sin 2y \sin y + \cos 2y \cos y}{\cos y}$ *B1M1*

$= \frac{\cos(2y - y)}{\cos y}$ *= 1 A1]*

[15]

12. (a) Dividing by $\cos^2\theta$: $\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} \equiv \frac{1}{\cos^2\theta}$ M1

Completion: $1 + \tan^2\theta \equiv \sec^2\theta$ A1 2

(no errors seen)

	(b)	use of $1 + \tan^2\theta = \sec^2\theta$: $2(\sec^2\theta - 1) + \sec\theta = 1$	M1	
		$[2\sec^2\theta + \sec\theta - 3 = 0]$		
		Factorising or solving: $(2\sec\theta + 3)(\sec\theta - 1) = 0$		
		$[\sec\theta = -\frac{3}{2}$ or $\sec\theta = 1]$		
		$\theta = 0$	B1	
		$\cos\theta = -\frac{2}{3}$; $\theta_1 = 131.8°$	M1 A1	
		$\theta_2 = 228.2°$	A1 ft	6
		[A1ft for $\theta_2 = 360° - \theta_1$]		

[8]

13.	(a)	$\cos 2A = \cos^2 A - \sin^2 A$ (+ use of $\cos^2 A + \sin^2 A \equiv 1$)	M1	
		$= (1 - \sin^2 A)$; $-\sin^2 A = 1 - 2\sin^2 A$ (*)	A1	2
	(b)	$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta$; $-3(1 - 2\sin^2\theta) - 3\sin\theta + 3$	B1; M1	
		$\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$	M1	
		$\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*)	A1	4
	(c)	$4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
		Complete method for R (may be implied by correct answer)		
		$[R^2 = 4^2 + 6^2, R\sin\alpha = 4, R\cos\alpha = 6]$	M1	
		$R = \sqrt{52}$ or 7.21	A1	
		Complete method for α; $\alpha = 0.588$	M1 A1	4
		(allow 33.7°)		
	(d)	$\sin\theta(4\cos\theta + 6\sin\theta - 3) = 0$	M1	
		$\theta = 0$	B1	
		$\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160..$ (24.6°)	M1	
		$\theta + 0.588 = (0.4291), 2.7125$ [or $\theta + 33.7° = (24.6°), 155.4°$]	dM1	
		$\theta = 2.12$	cao A1	5

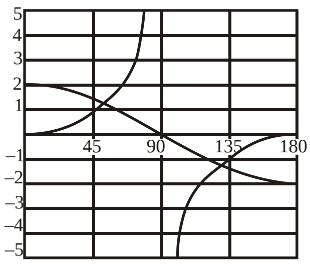
[15]

14.	(a)	Complete method for DE [e.g. split triangle ADE and sin, or sine or cos rule]	M1	
		$DE = 4\sin\theta$ (*) (c.s.o.)	A1(*)	2

(b)	$P = 2\,DE + 2EF$ or equivalent. With attempt at EF	M1	
	$= 8\sin\theta + 4\cos\theta$ (*) (c.s.o.)	A1(*)	2
(c)	$8\sin\theta + 4\cos\theta = R\sin(\theta + \alpha)$ $= R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$		
	Method for R, method for α *need to use tan for 2nd M*	M1 M1	
	[$R\cos\alpha = 8$, $R\sin\alpha = 4$ $\tan\alpha = 0.5$, $R = \sqrt{(8^2 + 4^2}$]		
	$R = 4\sqrt{5}$ or 8.94, $\alpha = 0.464$ (allow 26.6), *awrt 0.464*	A1 A1	4
(d)	Using candidate's $R\sin(\theta + \alpha) = 8.5$ to give $(\theta + \alpha) = \sin^{-1}\frac{8.5}{R}$	M1	
	Solving to give $\theta = \sin^{-1}\frac{8.5}{R} - \alpha$, $\theta = 0.791$ (allow 45.3)	M1 A1	
	Considering second angle: $\theta + \alpha = \pi$ (or 180) $- \sin^{-1}\frac{8.5}{R}$;	M1	
	$\theta = 1.42$ (allow 81.6)	A1	5

[13]

15. (a)



Tangent graph shape	M1	
180 indicated	A1	
Cosine graph shape	M1	
2 and 90 indicated	A1	4

Allow separate sketches.

(b)	Using $\tan x = \frac{\sin x}{\cos x}$ and multiplying both sides by $\cos x$. ($\sin x = 2\cos^2 x$)	M1	
	Using $\sin^2 x + \cos^2 x = 1$	M1	
	$2\sin^2 x + \sin x - 2 = 0$ (*)	A1	3

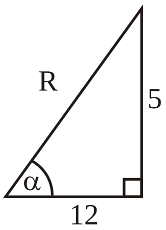
(c) Solving quadratic: $\sin x = \frac{-1 \pm \sqrt{17}}{4}$ (or equiv.) M1 A1

$x = 51.3$ (3 s.f. or better, 51.33…) α A1

$x = 128.7$ (accept 129) (3 s.f. or better) $180 - \alpha\ (\alpha \neq 90n)$ B1ft 4

[11]

16. (a) (i) $12\cos\theta - 5\sin\theta = R\cos\theta\cos\alpha - R\sin\theta\sin\alpha$.



$R^2 = 5^2 + 12^2, \Rightarrow \underline{R = 13}$ M1, A1

$\tan\alpha = \frac{5}{12}, \Rightarrow \alpha = 22.6°$ (AWRT 22.6)

or 0.39^C (AWRT 0.39^C) M1, A1 4

M1 for correct expression for R or R^2

M1 for correct trig expression for α

(b) (i) $\cos(\theta + 22.6) = \frac{4}{13}$ M1

$\theta + 22.6 = 72.1$, M1

$\underline{\theta = 49.5}$ (only) A1 3

M1 $\cos(\theta + \alpha) = \frac{4}{R}$

M1 $\theta + \alpha = \ldots$ *ft their R*

(ii) $\frac{8}{\tan\theta} - 3\tan\theta = 2$ M1

i.e. $0 = 3\tan^2\theta + 2\tan\theta - 8$ M1

$0 = (3\tan\theta - 4)(\tan\theta + 2)$ M1

$\tan\theta = \frac{4}{3}$ or -2

$\tan\theta = \frac{4}{3} \Rightarrow \underline{\theta = 53.1}$ A1

[*ignore θ not in range e.g. $\theta = 116.6$*] A1 5

M1 Use of $\cot\theta = \frac{1}{\tan\theta}$

M1 3TQ in $\tan\theta = 0$

M1 Attempt to solve 3TQ = 0

A1 For Final A mark must deal with $\tan\theta = -2$

[12]

17. (i) $\cos x \cos 30 - \sin x \sin 30 = 3(\cos x \cos 30 + \sin x \sin 30)$ M1

Correct use of cos(x ± 30)

$\Rightarrow \sqrt{3} \cos x - \sin x = 3\sqrt{3} \cos x + 3 \sin x$ M1, A1

Sub. for sin 30 etc
decimals M1, surds A1

i.e. $-4 \sin x = 2\sqrt{3} \cos x \to \tan x = -\dfrac{\sqrt{3}}{2}$ (*) M1, A1cso 5

Collect terms and use $\tan x = \dfrac{\sin x}{\cos x}$

(ii) (a) LHS = $\dfrac{1-(1-2\sin^2\theta)}{2\sin\theta\cos\theta}$ M1; A1

Use of cos 2A or sin 2A; both correct

$= \dfrac{\sin\theta}{\cos\theta} = \underline{\tan\theta}$ (*) A1 cso 3

(b) Verifying: 0 = 2 – 2 (since sin 360 = 0, cos 360 = 1) B1 cso 1

(c) Equation $\to 1 = \dfrac{2(1-\cos 2\theta)}{\sin 2\theta}$ M1

Rearrange to form $\dfrac{1-\cos 2\theta}{\sin 2\theta}$

$\Rightarrow \tan\theta = \frac{1}{2}$ or $\cot\theta = 2$ A1

i.e. θ = (26.6° or 206.6°) AWRT <u>27°, 207°</u>

1st solution M1

must be $\tan\theta = \pm\dfrac{1}{2}$ *or 2*

(both) A1 4

[13]

Alt 1

(c) $2 \sin\theta \cos\theta = 2 - 2(1 - 2\sin^2\theta)$ M1

Use of cos 2A <u>and</u> sin 2A

$0 = 2\sin\theta(2\sin\theta - \cos\theta)$

$\Rightarrow (\sin\theta = 0)\ \tan\theta = \frac{1}{2}$ etc, as in scheme A1

Alt 2

(c) $2\cos 2\theta + \sin 2\theta = 2 \Rightarrow \cos(2\theta - \alpha) = \dfrac{2}{\sqrt{5}}$ M1

$\alpha = 22.6$ (or 27) A1

$2\theta = 2\alpha$, 360, 360 + 2α

$\theta = \alpha$, 180 + α i.e. $\theta = 27°$ or 207° (or 1 dp)

$\theta = \alpha$ or 180 + α M1

A1 both

18. (a) $\sin x + \sqrt{3}\cos x = R\sin(x + \alpha)$

$= R(\sin x\cos\alpha + \cos x\sin\alpha)$ M1

$R\cos\alpha = 1$, $R\sin\alpha = \sqrt{3}$ A1

Method for R or α, e.g. $R = \sqrt{(1 + 3)}$ or $\tan\alpha = \sqrt{3}$ M1

Both $R = 2$ and $\alpha = 60$ A1 4

(b) $\sec x + \sqrt{3}\operatorname{cosec} x = 4 \Rightarrow \dfrac{1}{\cos x} + \dfrac{\sqrt{3}}{\sin x} = 4$ B1

$\Rightarrow \sin x + \sqrt{3}\cos x = 4\sin x\cos x$ M1

$= 2\sin 2x$ (*) M1 3

(c) Clearly producing $2\sin 2x = 2\sin(x + 60)$ A1 1

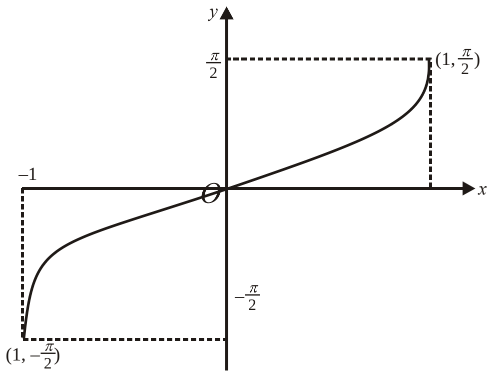
(d) $\sin 2x - \sin(x + 60) = 0 \Rightarrow \cos\dfrac{3x + 60}{2}\sin\dfrac{x - 60}{2} = 0$ M1

$\cos\dfrac{3x + 60}{2} = 0 \Rightarrow x = 40°, 160°$ M1 A1 A1 ft

$\sin\dfrac{x - 60}{2} = 0 \Rightarrow x = 60°$ B1 5

[13]

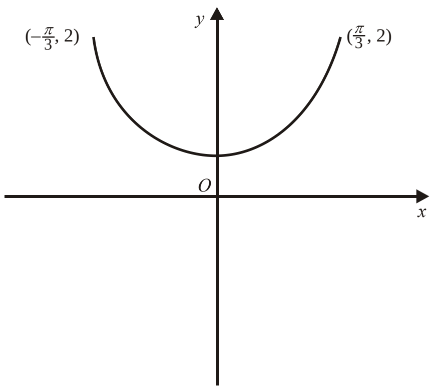
19. (a)



$y = \arcsin x$

(a) Shape correct
passing through O: G1;
end-points: G1 2

(b)



$y = \sec x$

Shape correct,
symmetry in Oy: G1
end-points: G1 2

(c)

x	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$
$\sec x$	2	1.155	1	1.155	2

Area estimate = $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec x \; dx = \frac{\pi}{6}\left[\frac{2+2}{2} + 1.155 + 1 + 1.155\right]$ M1 A1 A1

= 2.78 (2 d.p.) A1 4

[8]

20. (a) LHS = $\sin x + \sin 60° - \cos 60° \sin x$ M1

$= \sin x + \frac{\sqrt{3}}{2}\cos x - \frac{1}{2} \sin x$ A1

RHS = $\sin 60° \cos x + \cos 60° \sin x$ M1

		$= \dfrac{\sqrt{3}}{2}\cos x + \dfrac{1}{2}\sin x = \text{LHS}$	A1	4
	(b)	From (*a*), $\sin(60° + x) - \sin(60° - x) = \sin x$		
		$x = 24° \Rightarrow \sin 84° - \sin 36° = \sin 24°$	M1	
		$\Rightarrow \alpha = 24°$	A1	2
	(c)	$3\sin 2x + \sin 2x + \sin(60° - 2x) = \sin(60° + 2x) - 1$	M1	
		Using (*a*), $3\sin 2x = -1$	A1	
		$2x = 199.47°$ or $340.53°$	M1	
		$x = 99.7°, 170.3°$	A1	
		or $279.7°, 350.3°$	A1 ft	5

[11]

21.	(a)	$e^w = 0.25 \quad \Rightarrow \quad w = -1.39$	M1 A1	2
	(b)	$\arctan x = 1 \quad \Rightarrow \quad x = 0.79$	M1 A1	2
	(c)	$\ln \dfrac{y+1}{y} = 0.85 \quad \Rightarrow \quad \dfrac{y+1}{y} = e^{0.85}$	M1 A1	
		$\dfrac{1}{y} = 2.340 - 1 \quad \Rightarrow \quad y = 0.75$	M1 A1	4
	(d)	Putting $\cos z + \sin z$ in the form $\sqrt{2}\cos\left(z - \dfrac{\pi}{4}\right)$ or equivalent	M1 A1	
		$\cos\left(z - \dfrac{\pi}{4}\right) = \dfrac{1}{3\sqrt{2}}$		
		attempt for z	M1	
		$z = 2.12, -0.55$	A1, A1 ft	5

[13]

22.	(a)	$4\sin\theta - 3\cos\theta = R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$		
		$\sin\theta$ terms give $4 = R\cos\alpha$		
		$\cos\theta$ terms give $3 = R\sin\alpha$		
		$\tan\alpha = 0.75$	M1	
		$\alpha = 36.9^{\circ}$	A1	
		$R^2 = 4^2 + 3^2 = 25 \Rightarrow R = 5$	M1 A1	4

(b)	$5 \sin (\theta - 36.9^o) = 3$		
	$\sin (\theta - 36.9^o) = 0.6$	M1	
	$\theta - 36.9^o = 36.9^o, 143.1$	A1 M1	
	$\theta = 73.7^o, 180°$		
	awrt 74°	A1 A15	
(c)	Max value 5	B1	1
(d)	$\sin (\theta – 36.9^o) = 1$	M1	
	$\theta - 36.9° = 90°$		
	$\theta = 90° + 36.9^{\circ} = 126.9^{\circ}$	A1	2

[12]

1. This question was tackled with confidence by most candidates, many of whom gained at least 8 out of the 9 marks available.

In part (a), almost all candidates were able to obtain the correct value of R although $3^2 + 5^2 = 36$ was a common error for a few candidates, as was the use of the "subtraction" form of Pythagoras. A minority of candidates used their value of α to find R. Some candidates incorrectly wrote tanα as either $\frac{5}{3}$, $-\frac{3}{5}$ or $-\frac{5}{3}$. In all of these cases, such candidates lost the final accuracy mark for this part. A significant number of candidates found α in degrees, although many of them converted their answer into radians.

Many candidates who were successful in part (a) were usually able to make progress with part (b) and used a correct method to find the first angle. A significant minority of candidates struggled to apply a correct method in order to find their second angle. These candidates usually applied an incorrect method of (2π – their 0.27) or (2π – their α – their 0.27), rather than applying the correct method of (2π – their principal angle – their α). Premature rounding caused a significant number of candidates to lose at least 1 accuracy mark, notably with a solution of 0.28^c instead of 0.27^c.

2. Weaker candidates struggled with this unstructured trigonometry problem and therefore it was common for examiners to see many unsuccessful attempts at this question. A significant number of candidates either did not remember or were not able to derive the identity $\text{cosec}^2\theta = 1 + \cot^2\theta$. Some candidates could not use this identity to write down an identity for an angle of $2x$. Some candidates incorrectly deduced that $\text{cosec}^2 2x = 2 + \cot^2 2x$.

A significant minority of candidates decided to work in sines and cosines, but only a few of these managed to get beyond the equation $1 - \sin 2x \cos 2x = \sin^2 2x$. Some of these candidates then proceeded to use double angle formulae resulting in an extraordinarily complicated equation involving $\sin 4x$, which they struggled to solve.

Those candidates who obtained a quadratic equation in $\cot 2x$ were usually able to solve it to obtain the two values for $\cot 2x$ as 0 or 1. Virtually all of these candidates, however, thought that it was impossible for $\cot 2x = 0$ to have any solutions. Those candidates who rewrote $\cot 2x = 0$ as $\frac{\cos 2x}{\sin 2x} = 0$ usually found both solutions of $x = 45°$ and $x = 225°$.

3. In part (a), the majority of candidates started with $\cos^2\theta + \sin^2\theta = 1$ and divided all terms by $\cos^2\theta$ and rearranged the resulting equation to give the correct result. A significant minority of candidates started with the RHS of $\sec^2\theta - 1$ to prove the LHS of $\tan^2\theta$ by using both $\sec^2\theta = \frac{1}{\cos^2\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$. There were a few candidates, however, who used more elaborate and less efficient methods to give the correct proof.

In part (b), most candidates used the result in part (a) to form and solve a quadratic equation in $\sec\theta$ and then proceeded to find 120° or both correct angles. Some candidates in addition to correctly solving $\sec\theta = -2$ found extra solutions by attempting to solve $\sec\theta = \frac{2}{3}$, usually by proceeding to write $\cos\theta = \frac{2}{3}$, leading to one or two additional incorrect solutions. A significant minority of candidates, however, struggled or did not attempt to solve $\sec\theta = -2$.

A significant minority of candidates used $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and $\sin^2\theta = 1 - \cos^2\theta$ to achieve both answers by a longer method but some of these candidates made errors in multiplying both sides of their equation by $\cos^2\theta$.

4. The majority of candidates were able to give a correct proof in part (a). A number of candidates having written $\cos 2A = \cos^2 A - \sin^2 A$ did not make the connection with $\sin^2 A + \cos^2 A = 1$ and were unable to arrive at the given result.

Part (b) proved to be one of the most challenging parts of the paper with many candidates just gaining the first mark for this part by eliminating y correctly. A number of candidates spotted the link with part (a) and either substituted $\dfrac{1 - \cos 2x}{2}$ for $\sin^2 x$ or $1 - \cos^2 x$ for $2\sin^2 x$ and usually completed the proof in a few lines. A significant number of candidates manipulated $4\sin^2 x - 2\cos^2 x$ to $8\sin^2 x - 2$ and usually failed to progress further. There were some candidates who arrived at the correct result usually after a few attempts or via a tortuous route.

Part (c) was well done. R was usually correctly stated by the vast majority of candidates. Some candidates gave α to 1 decimal place instead of the 2 decimal places required in the question. Other candidates incorrectly wrote $\tan\alpha$ *as* $\frac{4}{3}$. In both cases, such candidates lost the final accuracy mark for this part. There was some confusion between $2x$ and α, leading to some candidates writing $\tan 2x$ *as* $\frac{3}{4}$ and thereby losing the two marks for finding α

Many candidates who were successful in part (c) were usually able to make progress with part (d) and used a correct method to find the first angle. A number of candidates struggled to apply a correct method in order to find their second angle. A significant number of candidates lost the final accuracy mark owing to incorrect rounding errors with either one or both of 51.7° or 165.3° seen without a more accurate value given first.

5. In part (a), most candidates were able to write down the correct identity for $\sin 2x$.

In part (b), there was a failure by a significant number of candidates who replaced $\operatorname{cosec} x$ with $\dfrac{1}{\sin x}$ to realise the connection between part (a) and part (b) and thus managed only to proceed as far as $1 = 8\sin x \cos x$. Some candidates, however, thought that $8\sin x \cos x$ could be written $\sin 8x$, presumably by continuing the imagined "pattern" with $2\sin x \cos x = \sin 2x$. Nonetheless, the majority of candidates who reached this stage usually used the identity in part (a) to substitute $4\sin 2x$ for $8\sin x \cos x$ and proceeded to give at least one allowable value for x.

A number of candidates lost the final accuracy mark for only giving one instead of two values for x, or for rounding one of their answers in radians incorrectly (usually by writing 1.45 instead of 1.44). Several candidates lost the final accuracy mark for writing their answers in degrees rather than radians. Some candidates, however, worked in degrees and converted their final answers to radians.

6. Part (a)(i) was well done and majority of candidates produced efficient proofs. Some candidates, however, failed to gain full marks when the incorrect use of, or omission of, brackets led to incorrect manipulation. Those who failed to spot the connection between parts (a)(i) and (a)(ii) rarely made any progress. Those who did make the connection often made sign errors and the incorrect equation sin $3\theta = -\frac{1}{2}$ was commonly seen. The majority of those who obtained the correct sin $3\theta = \frac{1}{2}$ did obtain the two answers in the appropriate range and the instruction to give the answers in terms of π was well observed.

Many candidates struggled with part (b) and, despite the hint in the question, blank responses were quite common. Those who did attempt to write 15° as the difference of two angles often chose an inappropriate pair of angles, such as 75° and 60°, which often led to a circular argument. If an appropriate pair of angles were chosen, those who used sin 45° cos 45° = $\frac{\sqrt{2}}{2}$ usually found it easier to complete the question than those who used sin45° cos 45° = $\frac{1}{\sqrt{2}}$.

7. This was the best done question on the paper and full marks were very common. There were a great many concise solutions to part (a), the great majority started from $\sin^2 \theta + \cos^2 \theta = 1$ and divided all the terms by $\sin^2 \theta$. Those who started from $1 + \cot^2 \theta$ often produced longer solutions but both marks were usually gained. The majority of candidates saw the connection between parts (a) and (b) and usually obtained both correct solutions. Candidates who substituted $\tan \theta = \frac{\sin \theta}{\cos \theta}$ could achieve the same results by a longer method but sometimes made errors in multiplying their equation by $\sin^2 \theta$. A significant number who used a quadratic in cosec θ obtained cosec $\theta = 5$ and got no further, seemingly deciding that it was not possible to solve this.

8. In part (a) most candidates took the given identity, divided by $\sin^2 \theta$ and correctly manipulated their equation to obtain the required result. Correct solutions were also given by those who started with the expression cosec $^2\theta$–cot$^2 \theta$ and used the given identity to show that this expression came to 1. However, those candidates who assumed the result (i.e.cosec $^2\theta$– cot$^2 \theta \equiv$ 1) and manipulated this to obtain the given identity were not given the final mark unless they drew at least a minimal conclusion from this (e.g. hence result). Candidates who understood the link between parts (a) and (b) and used the difference of two squares completed part (b) easily. Other, more lengthy, solutions were also seen. Weaker candidates tended to produce circular arguments or use incorrect statements such as cosec $^2\theta$– cot$^2 \theta$= 1 $\Rightarrow$ cosec $^4\theta$– cot$^4 \theta$= 1. The first method mark for linking parts (b) and (c) was gained by most candidates. Many were also able to use the result in part (a) to obtain a quadratic in cotθ Candidates who did not spot these links were usually unsuccessful. For those candidates who obtained a quadratic equation, factorising was generally done well although less proficiency was seen in giving solutions to the resulting trigonometric equations in the correct range.

9. Pure Mathematics P2

In (ai) most candidates identified the appropriate form of the double angle formula to substitute in the left hand side of this identity, but the following cancelling was frequently incorrect. The most popular (and incorrect) answer was:

$\frac{\cos 2x}{\cos x + \sin x} = \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \frac{\cos^2 x}{\cos x} - \frac{\sin^2 x}{\sin x} = \cos x - \sin x$. Some candidates started by multiplying both sides of the identity by ($\cos x + \sin x$) and multiplying out the difference of two squares. This method was acceptable provided that the candidate clearly picked out this form of the double angle formula as a known result and completed their argument correctly. The response to (aii) was usually much more successful, but many candidates clearly have problems over the distinctions between $\cos^2 x$, $\cos 2x$ and $2\cos x$.

In (b) most candidates started by applying (ai), but usually went on to pick out the double angle formulae in their working rather than apply (aii).

In the final part, some candidates expended considerable time and effort in trying to expand both sides of this equation to obtain an equation in a single trig.function. This invariably involved a false step of the form $\sin 2x = 2\sin x$. Some tried squaring both sides of the equation which allowed them an equation in $\sin 2x$ or $\cos 2x$, but also introduced false solutions that were not discarded. Those candidates who did realise that this equation is equivalent to an equation in $\tan 2x$ often went on to obtain the correct answers, but common errors here involved answers in degrees, radians but not expressed as multiples of π, or an incomplete set of solutions. Some candidates had difficulty in rewriting the equation and arrived at the false equation $\tan 2x = 0$.

Core Mathematics C3

Candidates who gained both the marks in part (a)(i) were in the minority. Most realised that the identity $\cos 2x \equiv \cos^2 x - \sin^2 x$ was the appropriate identity to use but the false working $\frac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \cos x - \sin x$ was as common as the correct working using the difference of two squares. Part (a)(ii) was better done and many gained full marks here. An unexpected aspect of many of the proofs seen here, and in part (b), was that the formula $\cos 2x \equiv \cos^2 x - \sin^2 x$ seemed to be much better known than $\cos 2x \equiv \cos^2 x - x - 1$. Many produced correct proofs using the former of these versions of the double angle formulae and $\cos 2x + \sin^2 x = 1$. This was, of course, accepted for full marks but did waste a little time and these times can mount up in the course of a paper. There were many correct proofs to part (b). The majority started using part (a)(i) but then finished their demonstrations using double angle formulae rather than part (a)(ii). Part (d) was clearly unexpected in this context. The topic is in the C2 specification and may be tested on this paper. Among those who did realise they could divide by $\cos 2\theta$, the error $\tan 2\theta = 0$ was common. Some very complicated methods of solution were seen but these were rarely successful. An error of logic was frequently. On reaching an equation of the form $f(\theta)\, g(\theta) = A$. where A is a non-zero constant, candidates proceeded to deduce that either $f(\theta) = A$ or $g(\theta) = A$. A few candidates drew diagrams showing clearly the symmetrical nature of $\cos 2\theta$ and $\sin 2\theta$ and deduced the four solutions from this. Such an approach is sound and was awarded full marks.

10. Part (a) was well done. Such errors as were seen arose from $\alpha = \pm 3$ or $\tan\alpha = -\frac{1}{3}$. In part (b), the majority could find the answer 38.0 although a substantial minority, with the correct method, failed to obtain this result through premature approximation. If an answer is required to one decimal place the candidate must work to at least 2 decimal places. The second answer proved more difficult. Many candidates produced their "secondary value" at the wrong place in their solution, giving the value of $360^\circ - \left(\arccos\frac{7}{\sqrt{160}} - \alpha\right) \approx 322.0^\circ$ instead of

$$360^\circ - \arccos\frac{7}{\sqrt{160}} - \alpha \approx 285.2^\circ .$$

Part (c) was not well done. In (c)(i) 12 or 0 were often given as the minimum value and few realised that the answer to (c)(ii) was the solution of $\cos(x + a) = -1$. Many produced solutions involving R. A few tried differentiation and this was rarely successful.

11. A few candidates offered no attempt at this question, but those who did attempt it seemed to score well.

(a) Many managed to expand the compound angles. Most seemed to be comfortable working with surds, and realised that tan□□θ was obtained by dividing □sinθ□by cosθ. A lot did get to a correct form for the answer, although there were many sign slips and errors in combining √3 and √3/2.

(b) (i) Many candidates produced very quick answers, although some could not progress beyond the initial statement $\cos 2\theta = \cos^2\theta - \sin^2\theta$

(ii) Most candidates made the link with the previous part and went on to form the required quadratic equation. Many seemed comfortable working in radians, although some worked in degrees and then converted their answers. Solutions outside the range were quite often seen, with $-\pi/2$ being particularly common. Several candidates reached the stage $1-2\sin^2 x = \sin x$, but could make no further progress. There were also several spurious attempts to solve this equation in this form. Some candidates ignored the instruction "hence". A few of these were able to deduce the correct values for x by considering sketches of the functions.

(iii) Many candidates produced very quick answers by looking at what they had shown in (b)(i), although in some cases after a page of work candidates had horrific expressions involving $\cot y$ and $\sec^2 y$. A number of candidates were not able to make progress because they did not attempt to rewrite $\tan y$. Some candidates resorted to showing that the relation was true for one of two specific values of y that they chose. There was a common misread in this part, with many candidates seeing $2y$ in place of $\sin 2y$.

12. (a) A variety of methods were used and most candidates were able to prove the identity. Many divided the given statement $\cos^2\theta + \sin^2\theta = 1$ by $\cos^2\theta$. Others started with the answer, replacing tan with sin/cos and sec with 1/cos. (Where an answer is printed it is important that each step of the argument is clearly shown to gain full credit.)

(b) Most candidates solved the trigonometric equation confidently, but many used sines and cosines rather than take the lead from the question and using sec. Factorisation was good and the solution $\theta = 0$ was usually found (but also 360, which was outside the range). For those who used the correct equation, the other two angles were found correctly. A number of candidates changed sec into $\sec^2$ at the start of this question, changing the question quite radically. There were not many who needed the final follow through mark, since most who reached that stage had the correct value for θ.

13. This question yielded the most variety of solutions: some showed good quality mathematics with concise methods and accurate answers.

(a) This was generally accessible to the candidates, but some could not move past the given identity.

(b) There were many correct answers to this part, but candidates did not always show sufficient working which is crucial when answers are given on the paper. There were some sign errors in the processing steps. Most candidates worked from left to right.

(c) The method here was generally well known but inaccuracies occurred and e.g. $R\sin\alpha = 6$ was too often seen. Some candidates failed to give their answers in this part correct to at least 3sf.

(d) Many candidates failed to notice the connection between b) and d). Not all candidates were able to solve the trig equation successfully, with $\sin\theta$ sometimes being lost rather than giving the solution $\theta = 0$. Those that did go on to solve the remaining equation were frequently unable to proceed beyond inverse sin to the correct answer 2.12. Some candidates attempted a squaring approach but generally ended up with excess solutions. Candidates did not always work in radians to 3 sf.

14. Parts (a) & (b) were poorly done with many candidates not getting (a) but working backwards to get (b). Much valuable time was wasted, often writing a page or more to gain 1 or 2 of the marks.

Part (c) was generally well done, with the main error being α being given in degrees rather the required radians. In Part (d) most candidates gained one value for x, but either did not work out the second one or incorrectly used π – first one. Accuracy marks were also lost in both (c) and then (d) by students not using accurate answers in follow through work.

15. Graph sketches in part (a) were often disappointing, with the tangent graph in particular proving difficult for many candidates. While some indicated the asymptote clearly, others seemed unsure of the increasing nature of the function or of the existence of a separate branch. Sketches of $y = 2\cos x$ were generally better, although some confused this with $y = \cos 2x$.

Part (b) was usually well done, with most candidates being aware of the required identities $\tan x = \dfrac{\sin x}{\cos x}$ and $\sin^2 x + \cos^2 x = 1$.

Although in part (c) some candidates produced a factorisation of the quadratic function, the majority used the quadratic formula correctly to solve the equation.

A few the omitted the second solution, but apart from this the main mistake was to approximate prematurely and then to give the answer to an inappropriate degree of accuracy (e.g. sin x = 0.78, therefore x = 51.26).

16. While many candidates demonstrated a clear understanding of the method required for part (i)(a), there was widespread confusion over what to do with the minus sign, and far too many candidates who were happy with statements such as $\sin\alpha = 5$ and $\cos\alpha = 12$.

Those who obtained 13 and 22.6 in part (a) usually went on to solve part (b) successfully. In (ii) most knew a correct definition for $\cot\theta$, but usually only those who formed a quadratic in $\tan\theta$ were able to solve the equation successfully. Here, and in part (b), the instruction to give answers to 1 decimal place was occasionally ignored.

There was some use of graphical calculators to solve part (ii). An answer of 53.1 can be obtained quite quickly this way, but this does not show that this is the only solution in the range and so no credit was given. Whilst candidates can use these calculators in P2 they should be encouraged to show their working in questions such as this if they wish to gain full credit.

17. The examiners were pleased with the quality of many responses to this question, in the past the trigonometry topic has not been answered well.

Most candidates were able to start part (i) correctly, and usually they then substituted for sin 30 and cos 30. There were a number of processing errors, lost minus signs or 2s, in the final stage to obtain tan x from $\dfrac{\sin x}{\cos x}$, but many fully correct solutions were seen.

In part (ii)(a) most knew the formula for sin 2x but those who started with $\cos 2x = \cos^2 x - \sin^2 x$ often failed to use brackets carefully and were unable to establish the result accurately. In part (b) most used the simple substitution as expected but some candidates tried to rearrange the equation and often this led to errors creeping in. The final part of the question discriminated well. The most successful solutions usually saw the link with part (a) and used $\dfrac{2(1-\cos 2\theta)}{\sin 2\theta} = 2\tan\theta$ but often this was put equal to 0, rather than 1, and no further progress was made. The stronger candidates sailed through all the parts of this question and produced some impressive solutions.

18. This question tested several aspects of this section of the syllabus and so it was good to see so many candidates confidently progress through the question until part (d).

In part (d) many candidates who correctly reached the stage $2\cos\left(\frac{3x+60}{2}\right)\sin\left(\frac{x-60}{2}\right) = 0$ seemed unable to continue, and others expanded at length producing copious amounts of irrelevant work. The important result that, if $pq = 0$ then either $p = 0$ or $q = 0$ seemed to be unfamiliar in this context. The minority of candidates who were familiar with the method went on to gain marks but it was rare to see all 5 marks earned in this part.

19. No Report available for this question.

20. No Report available for this question.

21. No Report available for this question.

22. No Report available for this question.